Nonparametric Components in Multivariate Discrete Choice Models

Marlene Müller

Institut für Statistik und Ökonometrie, Humboldt-Universität zu Berlin

marlene.mueller@gmx.de

http://ise.wiwi.hu-berlin.de/∼marlene

Plan

- Aim of the Analysis
- Multivariate Model
- Multivariate GEE
- Semiparametric Approach
- Conclusions

Aim of the Analysis

How do covariates for household i influence the possession of different assets?

$$
\begin{aligned}
Y_{i1} &= \mathbb{1}[\text{savings books}], \\
Y_{i2} &= \mathbb{1}[\text{stocks, bonds}], \\
Y_{i3} &= \mathbb{1}[\text{life insurance, private pensions}], \\
Y_{i4} &= \mathbb{1}[\text{own flat/ house}],
\end{aligned}
$$

where $\mathbb{1}$ denotes the indicator function. ($= 1$ if household i possesses the corresponding goods).

Data

The INSEE data set "Les Actifs Financiers 1991" covers data of 9530 French households. The about 4000 variables can be roughly grouped as:

"Actifs"	bank accounts, savings books, savings for future house ownership, stocks and bonds, life insurances and private pension plans, own flats/houses, grounds, professional goods (buildings, grounds, firms),
Household variables	household composition, age, education, profession, current occupation,
Ressources	income
Biography	socio-economic data on parents, heritages,
Bank credits	data on bank credits taken by the household.

	household types			
	(S)	**(C)**	**(S+K)**	**(C+K)**
	single	couple	single+children	family
Y_1 **savings book**	0.73	0.81	0.56	0.72
Y_3 **stocks & bonds**	0.25	0.36	0.08	0.22
Y_5 **insurance/pension**	0.37	0.50	0.35	0.46
Y_6 **real estate**	0.40	0.64	0.27	0.67
X_1 **female**	0.54	0.08	0.89	0.08
X_2 **Île de France**	0.24	0.18	0.23	0.14
X_3 **"cadres"**	0.20	0.29	0.07	0.20
X_4 **"intermediaires"**	0.23	0.25	0.25	0.24
X_5 **"employees"**	0.31	0.13	0.51	0.13
X_6 **age**	46.88 (14.52)	51.78 (14.27)	35.59 (5.65)	36.40 (5.63)
X_7 **income**	98666.77 (70631.16)	174098.00 (100252.14)	85215.88 (48924.68)	172339.81 (108547.09)
n	834	1102	168	1907

Table 1. Descriptive Statistics

Marginal Model

e.g. Probit

$$P(Y_i = 1|X = x_i) = E(Y_i|X = x_i) = \Phi(\eta(x_i))$$

where $\Phi(\bullet)$ denotes the Gaussian cdf

special case of **generalized linear model (GLM)**

$$E(Y_i|X = x_i) = G(\eta(x_i)), \quad \eta(x_i) = \beta^T x_i.$$

Likelihood & Quasi-Likelihood

likelihood $\Leftarrow$ exponential family

$$f(y_i, \theta_i, \phi) = \exp\left\{ \frac{y_i\theta_i - b(\theta_i)}{a(\phi)} - c(y_i, \phi) \right\}$$

maximum-likelihood

$$\sum_i \log f(Y_i, \theta_i, \phi) \to \max \iff \sum_i \theta_i' \left\{ Y_i - b'(\theta_i) \right\} \stackrel{!}{=} 0$$

$$\underset{\text{GLM}}{\iff} \sum_i D_i V^{-1}(\mu_i) \left\{ Y_i - \mu_i \right\} \stackrel{!}{=} 0$$

where in GLM $EY_i = \mu_i = G(\eta(x_i))$ and $Var(Y_i) = a(\phi)V(\mu_i)$ and D_i only depends on $G(\bullet)$, $\eta(\bullet)$ and x_i

Quasi-/Pseudo-Likelihood

$$EY_i = \mu_i, \quad Var(Y_i) = a(\phi)V(\mu_i)$$

$$\underset{\text{GLM}}{\Longleftrightarrow} \sum_i D_i V^{-1}(\mu_i)\{Y_i - \mu_i\} \stackrel{!}{=} 0$$

$\Rightarrow$ consistency, if $EY_i = \mu_i$ correctly specified

$\Rightarrow$ "Estimating Equation"

Generalized Estimating Equations

- if expectation $EY_i = G(\beta^T x_i)$ correctly specified, then the pseudo-ML is also consistent (Gourieroux, Monfort & Trognon 1984)
- in particular, consistency of $\widehat{\beta}$ is not influenced by misspecified $V(\bullet) \Rightarrow$ "Working Matrix"
- multivariate: "Generalized Estimating Equations" (GEE) (Liang & Zeger 1986, Zeger & Liang 1986)
- tom estimated variance/covariance structure consistently, expectation and variance/covariance need to be specified correctly

Multivariate Model

- conditional model:

$$E(Y_{ij}|Y_{ik,k\neq j}, X_{ij}) = f_j(Y_{ik,k\neq j}, X_{ij}),$$

  influence of X_{ij} might by hidden because of using $Y_{ik,k\neq j}$

- marginal model:

$$E(Y_{ij}|X_{ij}) = f_j(X_{ij}).$$

  covariance structure within the Y_{ij} needs to be estimated

- **here we consider**

$$E(Y_{ij}|X_i) = f_j(X_i),$$

  where $j = 1, \ldots, r$ (dimension) and $i = 1, \ldots, n$ (sample size)

Covariance structure

latent variable

$$Y_{ij} = \mathbb{1}[Y_{ij}^* > 0],$$

where Y_{ij}^* denotes the utility of household i to have asset j

model for Y_{ij}^*

$$Y_{ij}^* = \eta_j(X_i) + \varepsilon_{ij}$$

with

$$E\varepsilon_{ij} = 0, \quad Var(\varepsilon_{ij}) = 1, \quad \text{Corr}(\varepsilon_{ij}, \varepsilon_{ik}) = \rho_{jk}^*.$$

Association Measures

tetrachoric correlation between the Y^*:

$$\rho^*_{jk} = \text{Corr}(Y^*_{ij}, Y^*_{ik}),$$

alternatives

- correlation between the responses

$$\rho_{jk} = \text{Corr}(Y_{ij}, Y_{ik}),$$

- odds-ratio:

$$\omega_{jk} = \frac{P(Y_{ij} = 1, Y_{ik} = 1) \cdot P(Y_{ij} = 0, Y_{ik} = 0)}{P(Y_{ij} = 1, Y_{ik} = 0) \cdot P(Y_{ij} = 0, Y_{ik} = 1)}.$$

  independence $\rightarrow \omega_{jk} = 1$; $\log \omega_{jk} \in (-\infty, \infty)$.

Multivariate GEE

$$\sum_i \mathcal{D}_i^T \mathcal{V}_i^{-1} \mathcal{S}_i = 0, \qquad \text{where} \quad \mathcal{D}_i = \mathcal{J}_i \mathcal{X}_i, \quad \mathcal{J}_i = \text{diag}\left(\frac{\partial \mu_{ij}}{\partial \eta_{ij}}\right)$$

Fisher Scoring algorithm:

$$\begin{aligned} \beta_{new} &= \beta + \left(\sum_i \mathcal{D}_i^T \mathcal{V}_i^{-1} \mathcal{D}_i\right)^{-1} \sum_i \mathcal{D}_i^T \mathcal{V}_i^{-1} \mathcal{S}_i \\ &= \left(\sum_i \mathcal{X}_i^T \mathcal{W}_i \mathcal{X}_i\right)^{-1} \sum_i \mathcal{X}_i^T \mathcal{W}_i \mathcal{Z}_i \end{aligned}$$

with

$$\mathcal{W}_i = \mathcal{J}_i \mathcal{V}_i^{-1} \mathcal{J}_i, \quad \mathcal{Z}_i = \mathcal{X}_i \beta + \mathcal{J}_i^{-1} \mathcal{S}_i$$

Example: Bivariate GEE

Sei $\sigma_{i12} = \sigma(X_i, \beta_1, \beta_2, \gamma) = \mathsf{Cov}(Y_{i1}, Y_{i2})$, then

$$\sum_i \mathcal{D}_i^T \mathcal{V}_i^{-1} \mathcal{S}_i = 0$$

where

$$\mathcal{D}_i^T = \begin{pmatrix} \frac{\partial}{\partial \beta_1}\mu_{i1} & \frac{\partial}{\partial \beta_1}\mu_{i2} & \frac{\partial}{\partial \beta_1}\sigma_{i12} \\ \frac{\partial}{\partial \beta_2}\mu_{i1} & \frac{\partial}{\partial \beta_2}\mu_{i2} & \frac{\partial}{\partial \beta_2}\sigma_{i12} \\ 0 & 0 & \frac{\partial}{\partial \gamma}\sigma_{i12} \end{pmatrix},$$

and

$$\mathcal{S}_i = \begin{pmatrix} Y_{i1} - \mu_{i1} \\ Y_{i2} - \mu_{i2} \\ S_{i12} - \sigma_{i12} \end{pmatrix}, \quad S_{i12} = (Y_{i1} - \mu_{i1})(Y_{i2} - \mu_{i2}).$$

"Working" Covariance Structure

$$\mathcal{V}_i = a(\phi)\left\{\text{diag}(\,Var(\mathcal{S}_i))\right\}^{1/2}\mathcal{R}\left\{\text{diag}(\,Var(\mathcal{S}_i))\right\}^{1/2}$$

simplest model: independence $\Rightarrow \mathcal{R} = \mathcal{I}$

$$\mathcal{V}_i = a(\phi)\begin{pmatrix} V(\mu_{i1}) & 0 & 0 \\ 0 & V(\mu_{i2}) & 0 \\ 0 & 0 & V(\mu_{i1})V(\mu_{i2}) \end{pmatrix},$$

Fisher Scoring algorithm

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \gamma \end{pmatrix}_{new} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \gamma \end{pmatrix} + \left(\sum_i \mathcal{D}_i^T\mathcal{V}_i^{-1}\mathcal{D}_i\right)^{-1}\left(\sum_i \mathcal{D}_i^T\mathcal{V}_i^{-1}\mathcal{S}_i\right).$$

Probit GEE Correlations

	(S)	(C)	(S+K)	(C+K)
ρ^*_{12}	0.20	0.27	-0.28	0.20
ρ^*_{13}	0.26	0.09	-0.10	0.19
ρ^*_{14}	0.05	0.12	-0.06	0.05
ρ^*_{23}	0.43	0.29	0.00	0.25
ρ^*_{24}	0.16	0.15	0.48	0.05
ρ^*_{34}	0.10	0.14	0.11	-0.01

Table 2. Estimated correlations using probit for different household types.

Semiparametric Approach

$$E(Y|x,t) = G(\eta(x,t)), \quad G(\bullet) \text{ known (z.B. } G = \Phi)$$

- estimating GLM/GEE:
  weighted LS using the adjusted dependent variable z
  (Nelder & Wedderburn 1972)
- estimating a semiparametric GEE:
  semiparametric algorithm using the adjusted dependent variable z
  (cf. Wild & Yee (1996))

Generalized Linear Model (GLM)

$\star$ $\eta(x,t) = \alpha + \beta^T x + \delta^T t$

Generalized Partial Linear Model (GPLM)

$\star$ $\eta(x,t) = \beta^T x + m(t)$

Generalized Additive Model (GAM)

$\star$ $\eta(x,t) = \alpha + \beta^T x + \sum_{j=1}^{q} m_j(t_j)$

Household Portfolio Allocation

$n = 834$, Singles

$$\begin{aligned} Y_{i1} &= \mathbb{1}[\text{savings books}], \\ Y_{i2} &= \mathbb{1}[\text{stocks \& bonds}], \\ Y_{i3} &= \mathbb{1}[\text{insurance/pension}], \\ Y_{i4} &= \mathbb{1}[\text{real estates}], \end{aligned}$$

possible nonparametric components:

$$T_i = \text{age, income,}$$

linear covariables:

$$X_i = \text{female, Île de France, occupation}$$

Model to Estimate

marginal

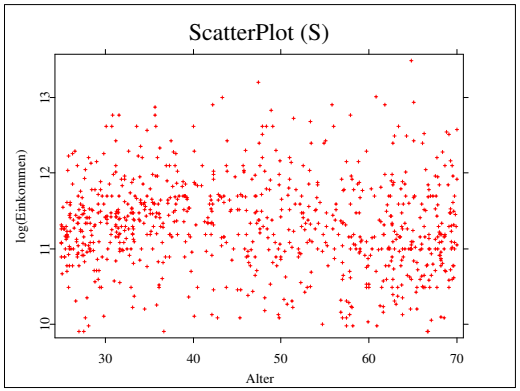
$$E(Y_{ij}|x_i, t_i) = P(Y_{ij} = 1|x_i, t_i) = \Phi\{\beta_j^T x_i + m_j(t_i)\}$$

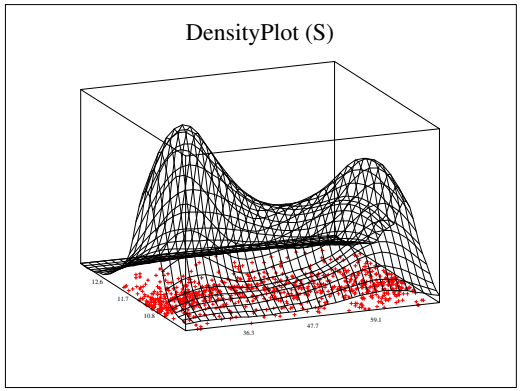
covariance structure = tetrachoric correlations

$$\rho^*_{jk} = \mathsf{Corr}(\varepsilon_{ij}, \varepsilon_{ik})$$

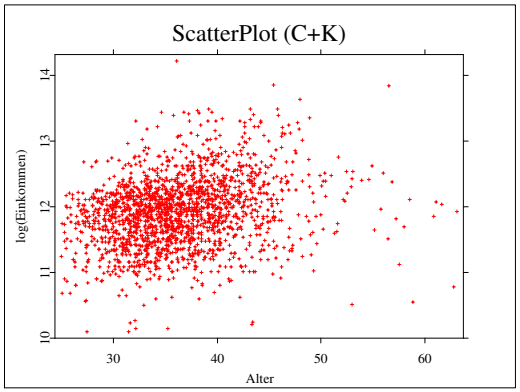
Exploring the Data

Singles (S)





Families (C+K)



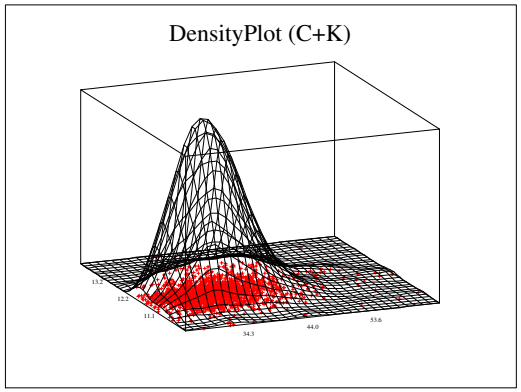


Figure 1. Scatterplot and bivariate density estimates for age and log(income).

Marginal Coefficients

e.g. singles (S) and savings books, $n = 834$

	Variable	**Linear**	**Part. Linear** $m(t_1)$	**Part. Linear** $m(t_1, t_2)$
Y_1 savings books	**intercept**	2.371 (1.97)	–	–
	female	0.295 (2.84)	0.292 (2.80)	0.285 (2.71)
	Île de France	-0.043 (-0.37)	-0.052 (-0.44)	-0.042 (-0.36)
	"cadres"	0.859 (4.49)	0.889 (4.53)	0.824 (4.43)
	"intermediaires"	0.403 (2.72)	0.339 (2.29)	0.340 (2.33)
	"employees"	0.138 (1.05)	0.109 (0.84)	0.113 (0.87)
	age	0.004 (1.22)	0.005 (1.52)	–
	log(income)	-0.211 (-1.96)	–	–

Table 3. Coefficients (t values) for probit fit and partial linear probit models.

Significant Effects

	(S)	(C)	(S+K)	(C+K)
Y_1 **savings books**	female occupation (income)			–female –Île income
Y_2 **stocks & bonds**	occupation age income	occupation age income		–Île occupation income
Y_3 **life insurance/pension**	female –Île (occupation) age income	age income	age	–Île (occupation) (income)
Y_4 **real estates**	–Île (occupation) age income	–Île age income	age (income)	–Île (–occupation) age income

Table 4. Significance for the coefficients of the parametric fit.

Nonparametric Curves

(S)

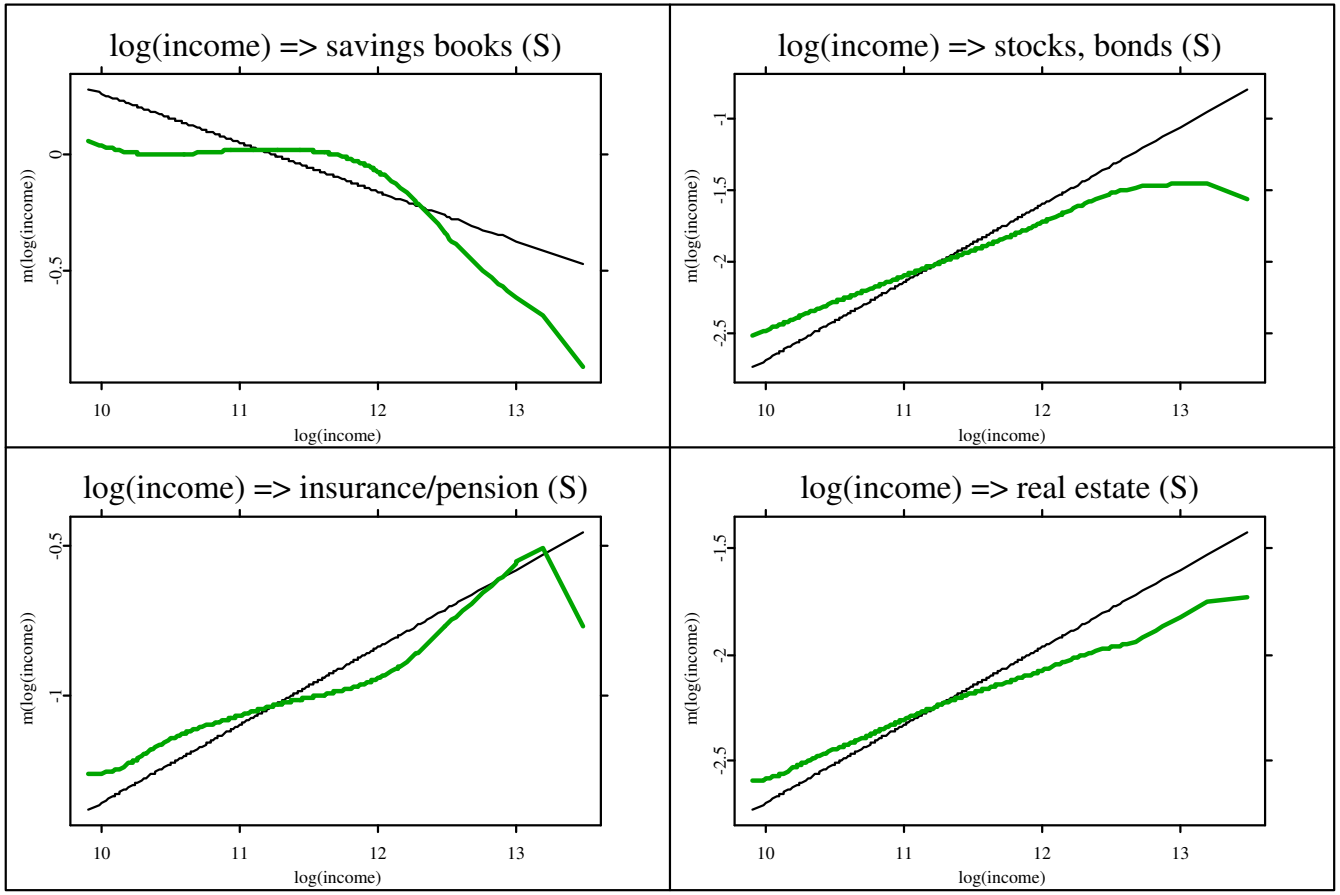


Figure 2. Effect of log(income). Nonparametric curves and linear fits.

(C+K)

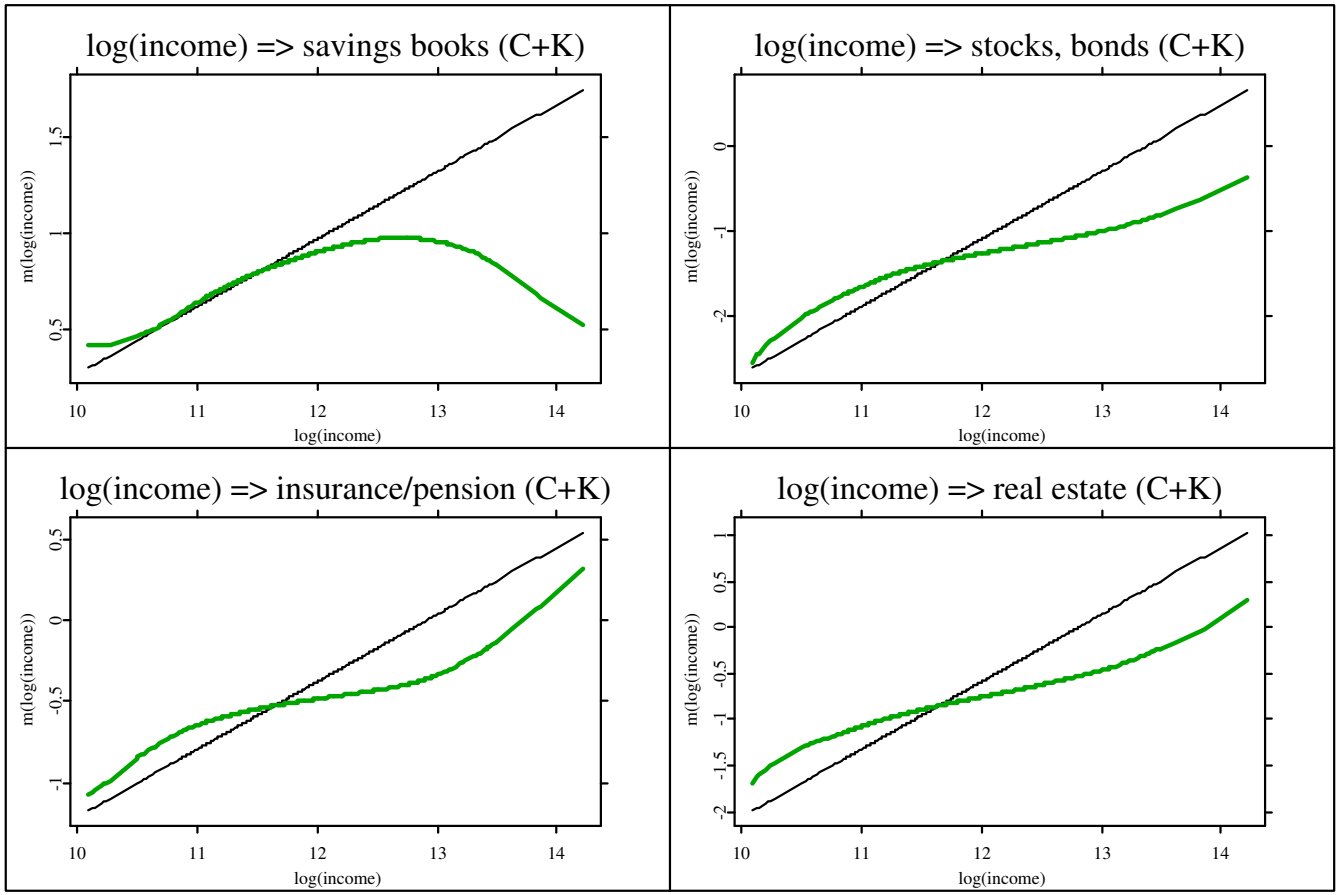


Figure 3. Effect of log(income). Nonparametric curves and linear fits.

Nonparametric Surfaces

(S)

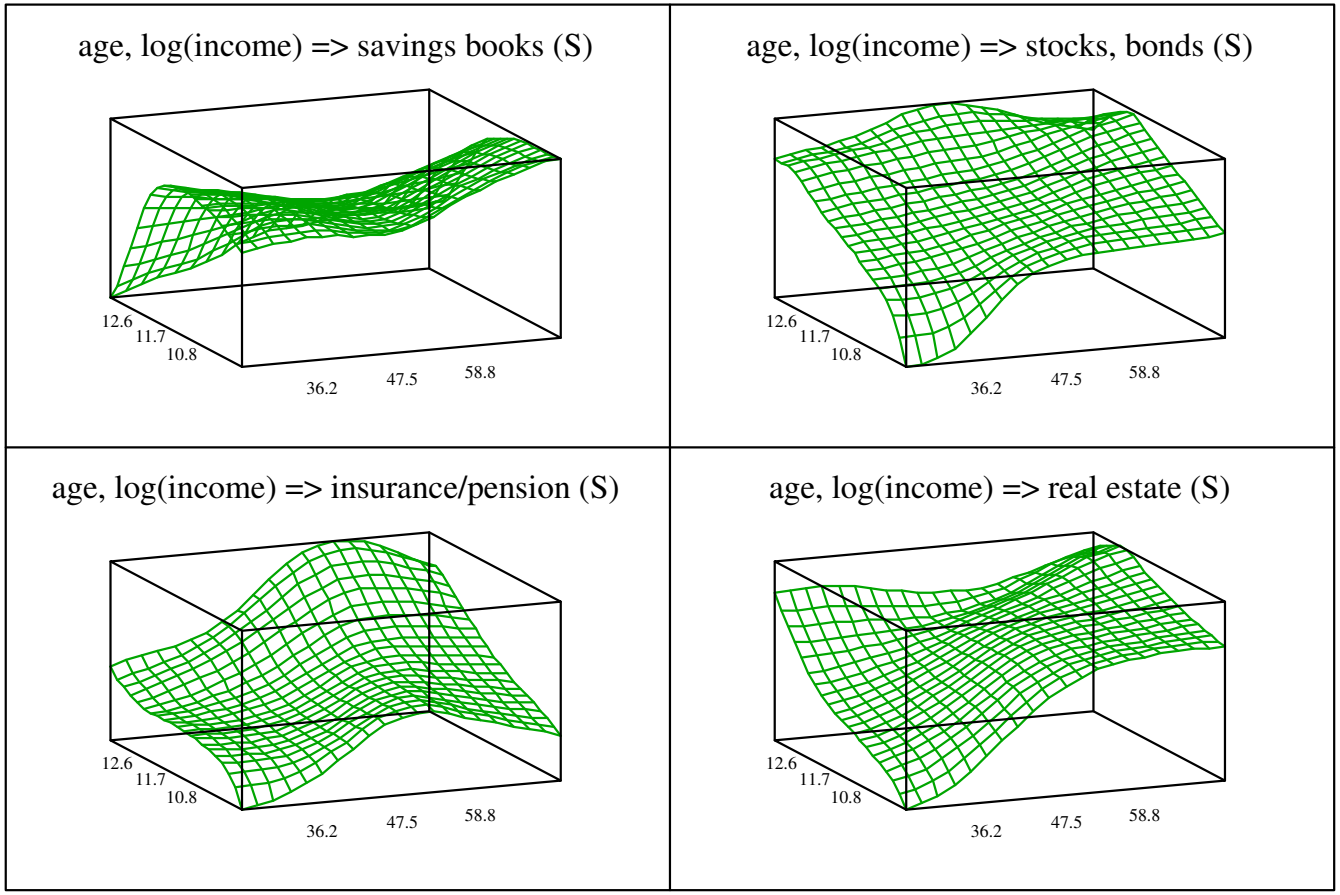


Figure 4. Effect of age and log(income).

(C+K)

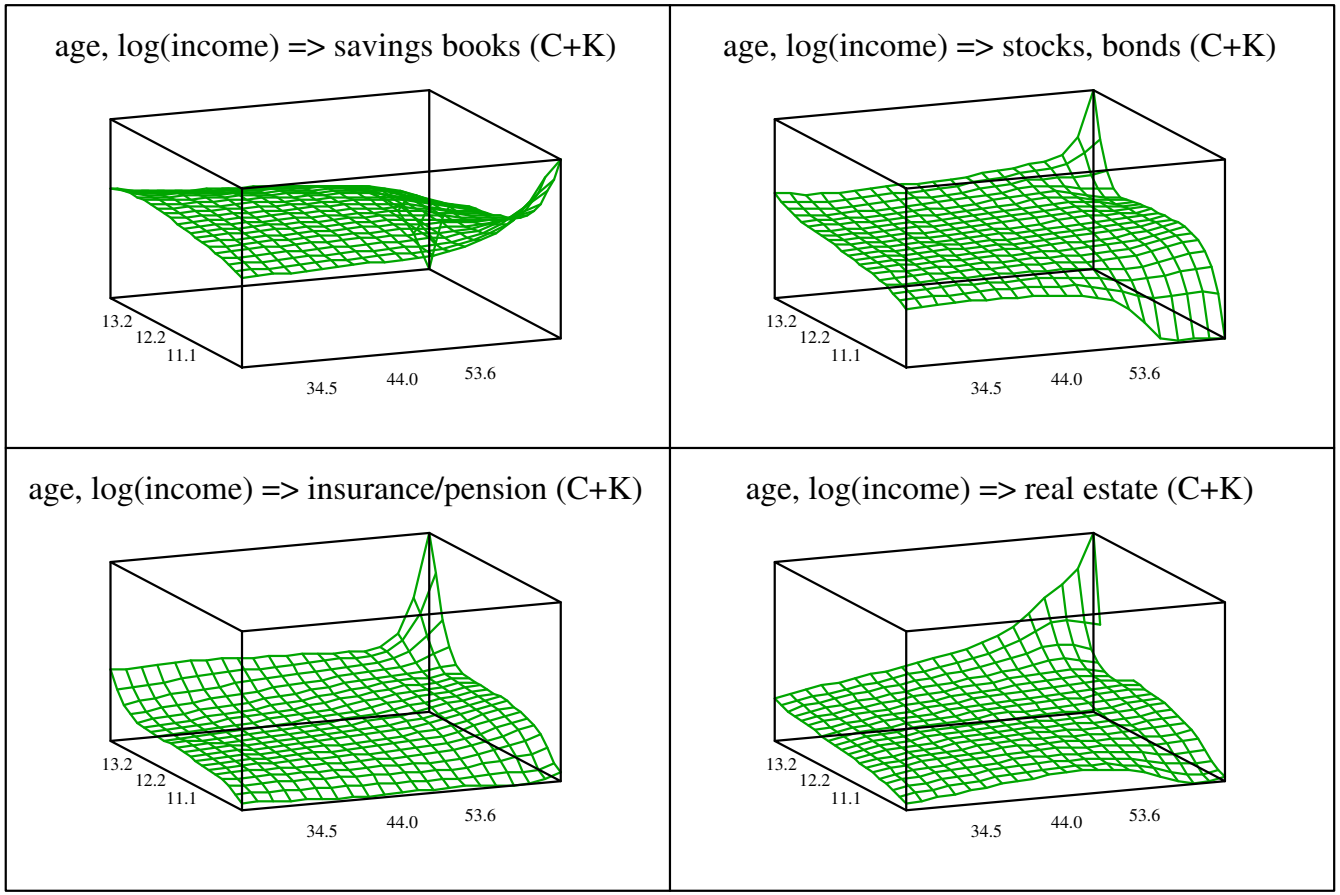


Figure 5. Effect of age and log(income).

Effect of age

(S)

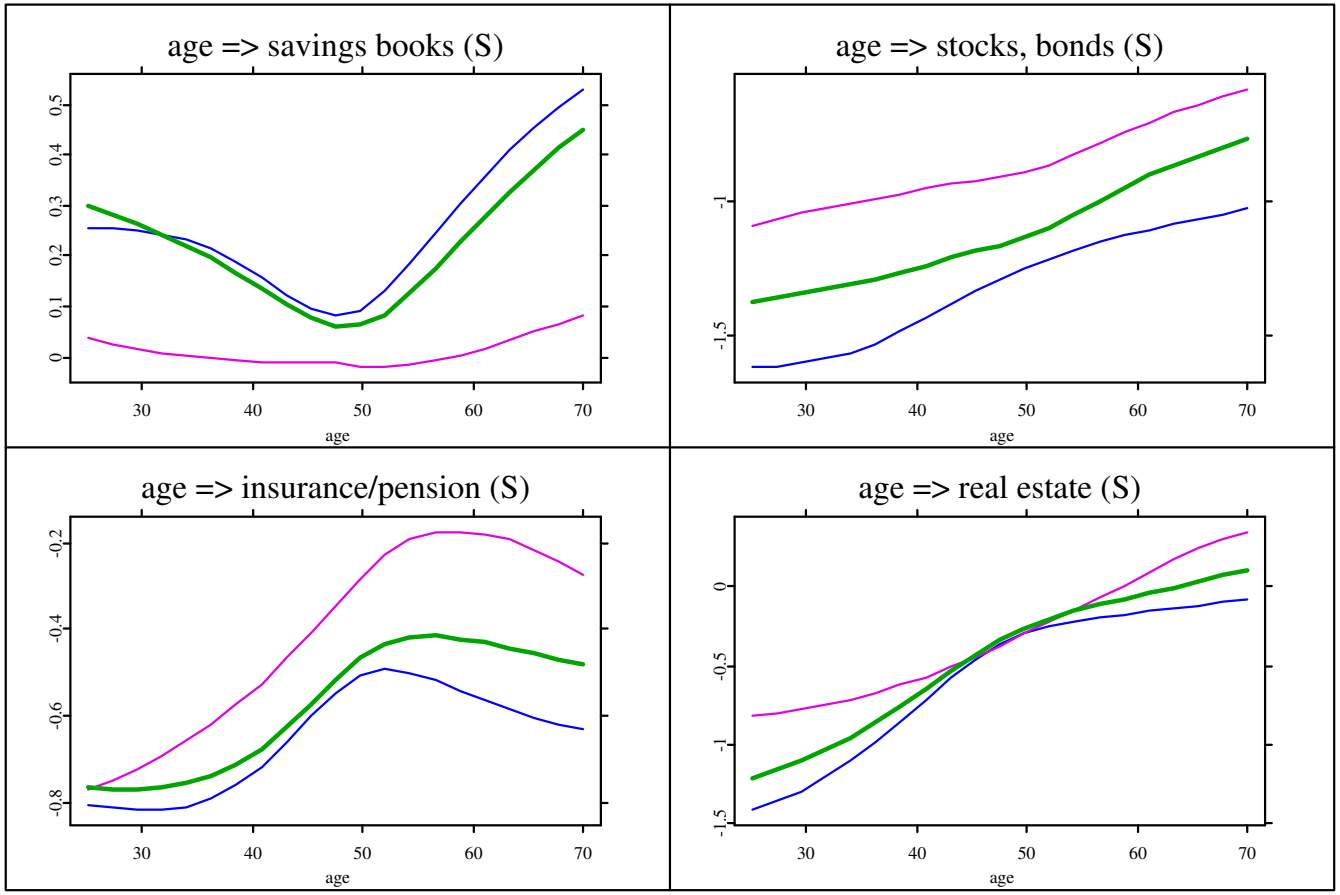


Figure 6. Effect of age. (Income fixed.)

(C+K)

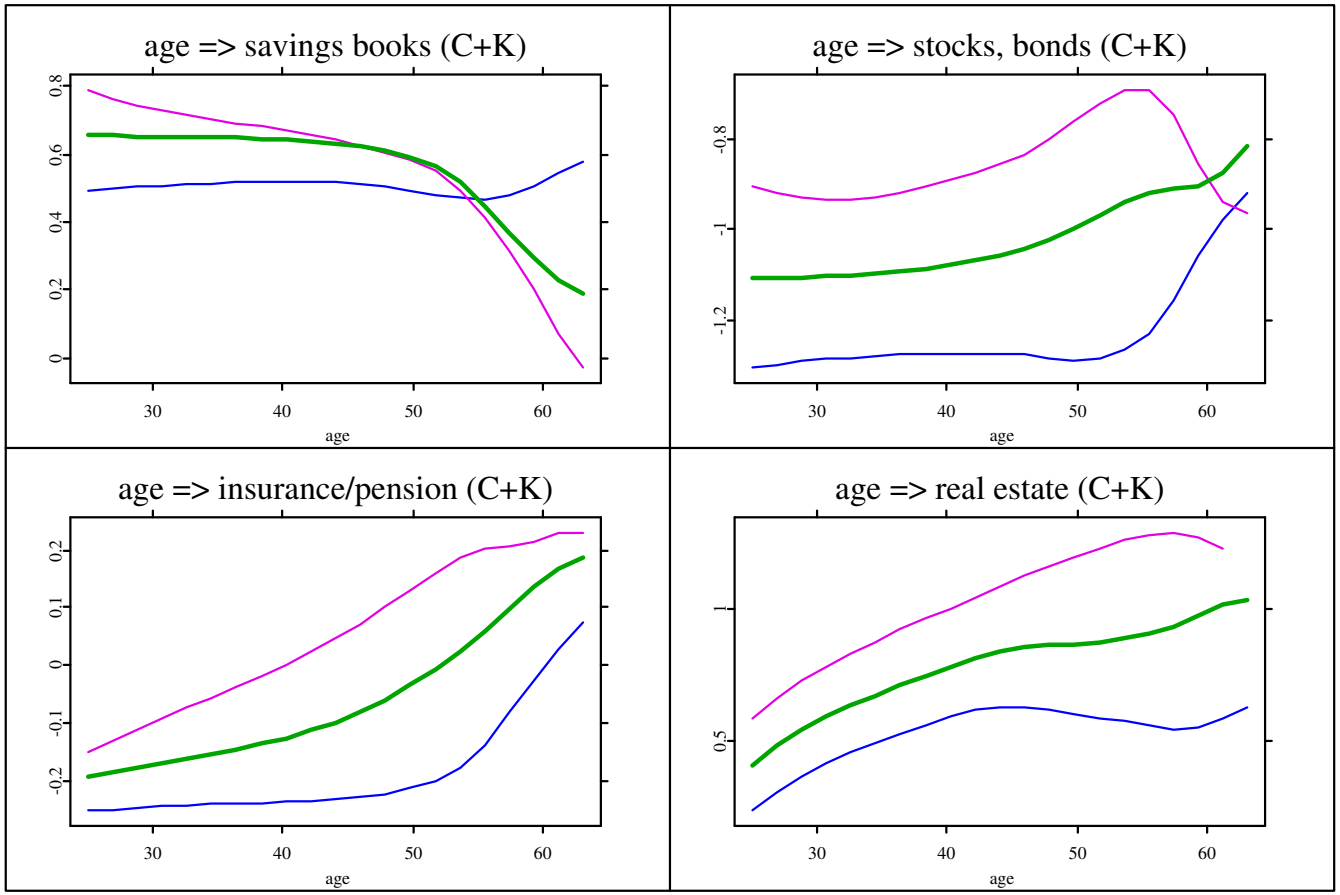


Figure 7. Effect of age. (Income fixed.)

Effect of log(income)

(S)

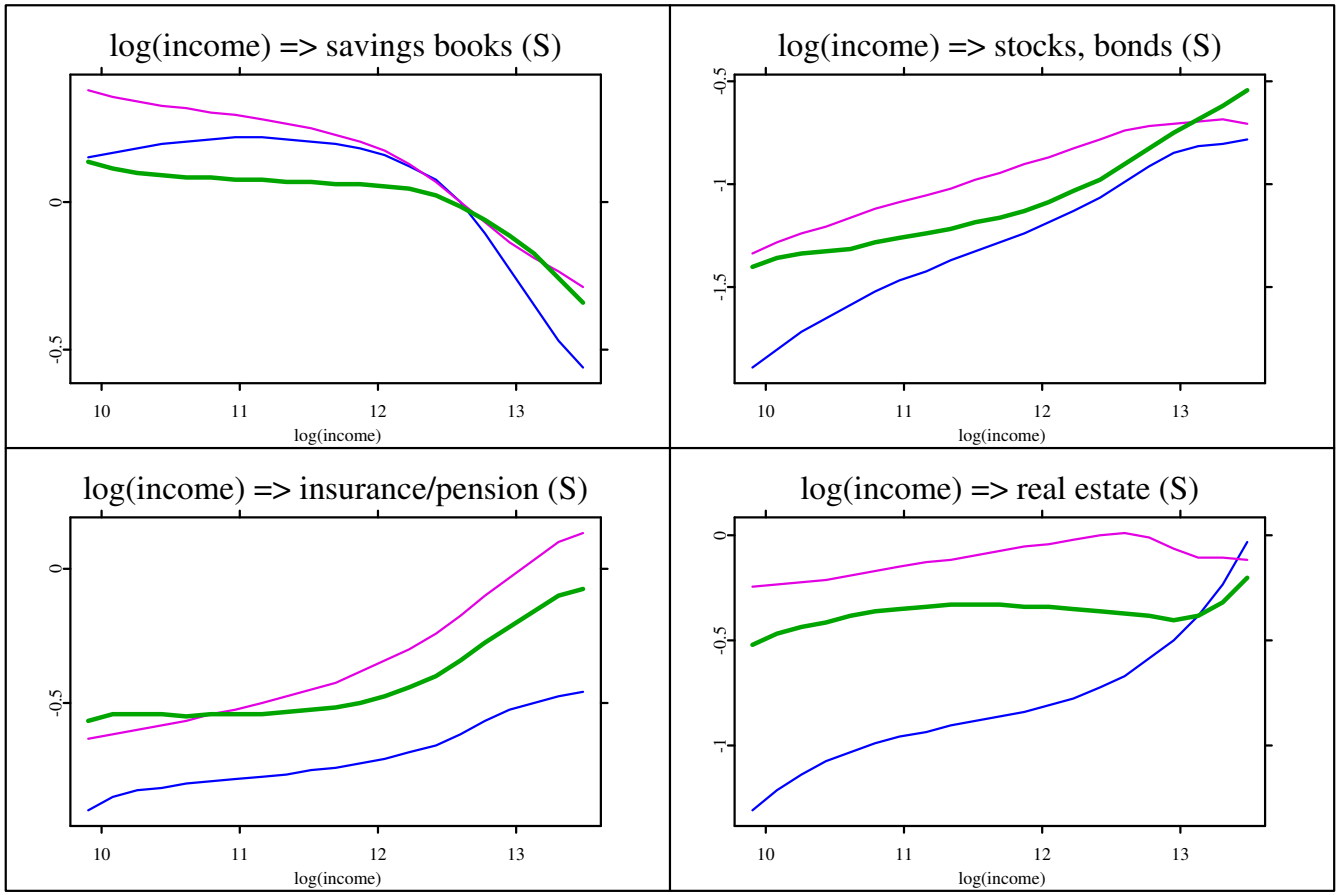


Figure 8. Effect of income. (Age fixed.)

(C+K)

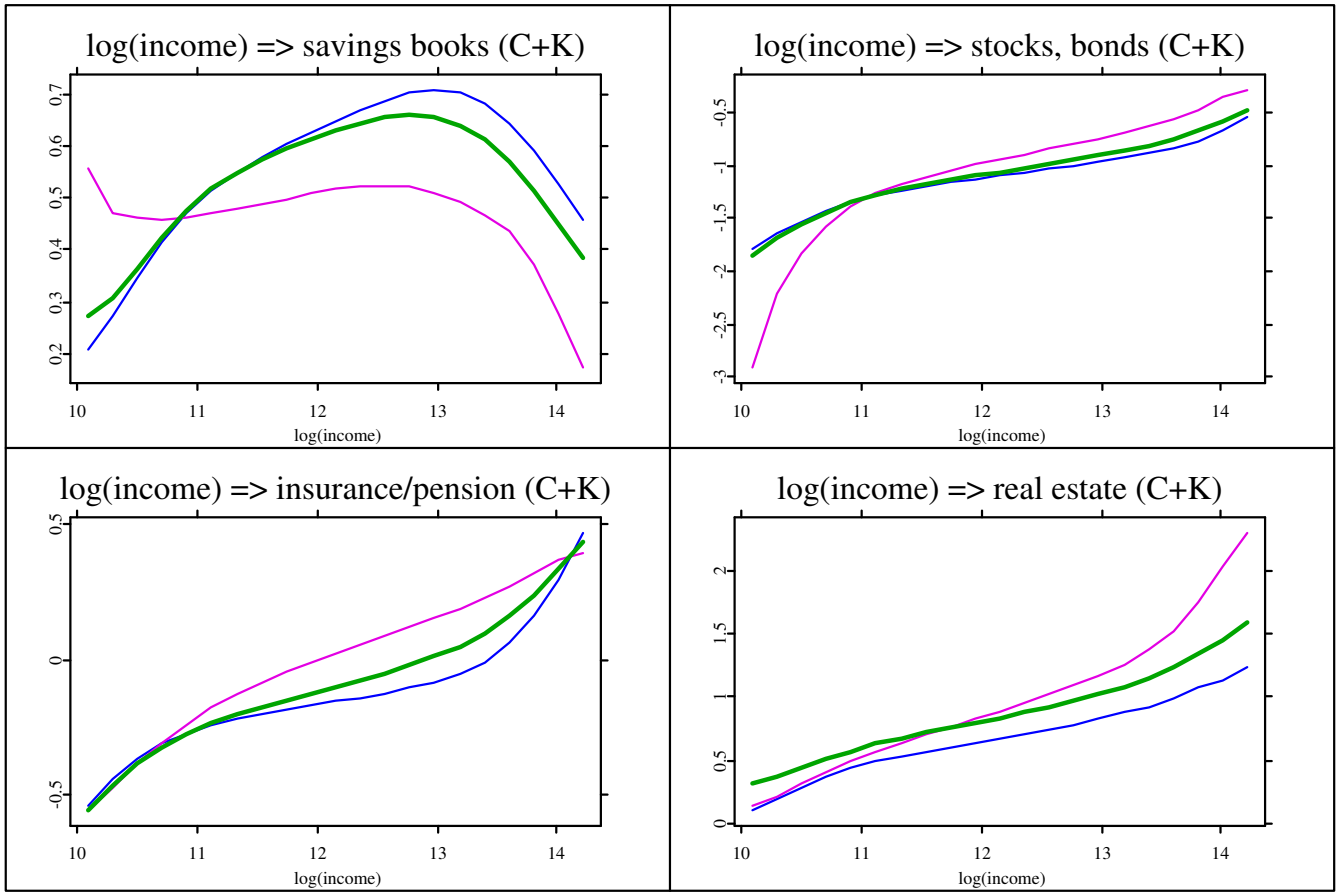


Figure 9. Effect of income. (Age fixed.)

Tetrachoric correlations

parametric

	(S)	(C)	(S+K)	(C+K)
ρ^*_{12}	0.20	0.27	-0.28	0.20
ρ^*_{13}	0.26	0.09	-0.10	0.19
ρ^*_{14}	0.05	0.12	-0.06	0.05
ρ^*_{23}	0.43	0.29	0.00	0.25
ρ^*_{24}	0.16	0.15	0.48	0.05
ρ^*_{34}	0.10	0.14	0.11	-0.01

semiparametric

	(S)	(C)	(S+K)	(C+K)
ρ^*_{12}	0.18	0.25	-0.30	0.20
ρ^*_{13}	0.28	0.12	-0.11	0.20
ρ^*_{14}	0.06	0.16	-0.06	0.08
ρ^*_{23}	0.42	0.31	-0.03	0.25
ρ^*_{24}	0.16	0.16	0.47	0.08
ρ^*_{34}	0.09	0.13	0.09	0.01

Table 5. Correlations for different household types.

Conclusion

- GEE permit the estimation of nonparametric components in multivariate generalized linear models
- a "working covariance matrix" reduces the computational effort
- outlook: semiparametric estimation of correlations

References

Fahrmeir, L. & Tutz, G. (1994). *Multivariate Statistical Modelling Based on Generalized Linear Models*, Springer.

Gourieroux, C., Monfort, A. & Trognon, A. (1984). Pseudomaximum likelihood methods: Theory, *Econometrica* **52**: 681–700.

Härdle, W., Mammen, E. & Müller, M. (1998). Testing Parametric versus Semiparametric Modelling in Generalized Linear Models. *Journal of the American Statistical Association*, **93**(444), 1461–1474.

Liang, K. Y. & Zeger, S. L. (1986). Longitudinal data analysis using generalized linear models, *Biometrika* **73**: 13–22.

Nelder, J. A. & Wedderburn, R. W. M. (1972). Generalized linear models, *Journal of the Royal Statistical Society, Series A* **135**(3): 370–384.

Prentice, R. L. (1988). Correlated binary regression with covariates specific to each binary observation, *Biometrics* **44**: 1033–1048.

Severini, T. A. & Staniswalis, J. G. (1994). Quasi-likelihood estimation in semiparametric models, *Journal of the American Statistical Association* **89**: 501–511.

Severini, T. A. & Wong, W. H. (1983). Generalized profile likelihood and conditionally parametric models, *Annals of Statistics* **20**: 1768–1802.

Wild, C. J. & Yee, T. W. (1996). Additive Extensions to Generalized Estimating Equation Methods, *Journal of the Royal Statistical Society, Series B* **58**: 711–725.

Zeger, S. L. & Liang, K. Y. (1986). Longitudinal data analysis for discrete and continuous outcomes, *Biometrics* **42**: 121–130.

Zhao, L. P. & Prentice, R. L. (1990). Correlated binary regression using a quadratic exponential model, *Biometrika* **77**: 642–648.